Transformation of Bend Test Material Data for Input to Finite Element Calculations

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Summary
The accurate prediction of contact spring characteristics with finite element programs requires the input of a stress-strain curve, for anisotropic materials in the rolling direction as well as transverse to the rolling direction. Usually the stress strain curve is only available from a tensile test in the rolling direction.
The method proposed in this paper is to determine the required data in an "as simple as possible" and well defined bend-test. A bend test is more similar to the practical application than a tensile test, it can be done on very small samples and in different directions, and a simple finite element simulation can be used for feed-back and verification. An optimised version of the "PD"-method test set-up [1] has been used for the measurements.
This paper describes and discusses the improved test, its results and the transformation from bend data to a stress-strain curve, and includes the feed-back through finite element simulations.

Key words:
Bend-test, Stress-strain curve, Anisotropy, Contact Spring, Finite element analysis.

1. Introduction

At the ICEC-conference in Sendai I presented a novel method to measure the elastic and plastic properties from a number of materials used for the fabrication of contact springs [1]. Important conclusions have been (a) that there is a strong anisotropy for steel and for several copper alloys, resulting in better spring properties in transverse direction, (b) that there is a strong difference between plasticity in forward and reverse bending and (c) that the set-up needed improvement for a better measurement of the thinner materials (<0.15 mm).
The measurement itself consists of taking a force-deflection curve by bending under well defined conditions. It enables a good comparison between materials mutually and provides data for use in finite element calculations. The improvement consists of making new tools with reduced distances between the pivot point and the clamping point: from 4.5 mm to 1, 2 and 3 mm at choice. The smaller distances are to be used with the thinner materials.
Test data are directly useful to compare materials but need some manipulation and calculation to produce input for finite element simulation. The material data required in finite element programs for spring simulation are a stress-strain curve and a Poisson constant. Two issues follow from this.
The first is to transform the measured force-deflection curves from the 10x10 mm test samples with various thicknesses to stress-strain curves for the particular materials.
The second is to measure the Poisson constant, maybe also in rolling direction and transverse. A possible method will be discussed, measurements are not yet done.
In previous paper [1] a stress σ_{pd} has been introduced to represent the maximum stress level in the bend test and it has been found that these values were considerably higher than the common ultimate strength σ_{uts} specified by the material suppliers. The theory of plasticity [2] of bending flat stock explains this very well, the value of σ_{pd} must be multiplied with a factor $\frac{1}{2}\sqrt{3}$ to be comparable with the σ_{uts} value in the tensile test, which is also equal to the maximum von Mises-stress $\sigma_{vonMises,max}$. Reduced with this factor $\frac{1}{2}\sqrt{3}$ the σ_{pd} values are when well in line with the specified values by material suppliers.
Several other papers describe [3] [4] and discuss [5] standard tests and express the need for feed-back by simulation [6] [7]. In this paper the measurement method and the transformation from force-deflection curves to stress-strain curves will be discussed. Measurement data from the PD bend test with 4 out of the 17 tested materials will be presented and used to explain different approaches for feed-back via finite element simulation.

2. The PD bend test and the 3-point bend test

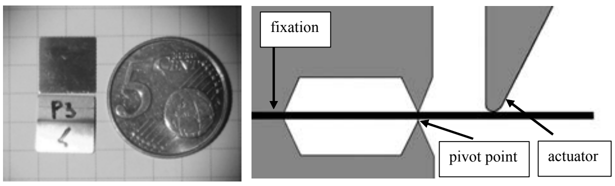

Fig. 1. A virgin and a tested sample compared with a 5 €ct coin in left picture and at right the test sample seen from aside in the measurement position. From left to right fixation, pivot point and actuator.

In the PD bend test a 10x10 mm test sample with a thickness in the 0.1-0.4 mm range is bent over a small angle, figure 1 shows the test samples and test set-up.

The elastic and plastic material properties can then be derived from the force-deflection curve.
Simple linear formulas can be used to derive values for the E-Modulus E and for the maximum stress σ_{uts}.
The measurement set-up has been described previously in more detail in [1] and is shown in figure 2 and 3 together with the mechanical scheme. The measurement can in principle also be done with a 3-point bending test (3P-test), provided that the measurement length must be adjustable to adapt to the thickness of the sample. Figure 4 shows the mechanical schemes of the two methods.
An important aspect in these measurements is to choose the length between load, pivot and fixing points to be longer than 10x the material thickness and at the same time short enough to achieve plastic deformation at small deflections. The latter is crucial to limit deviations due to friction and change of geometry.
A practical challenge is to have a perfect alignment of sample and tool. That is the background why the PD-test is preferred over the 3-P bending test.

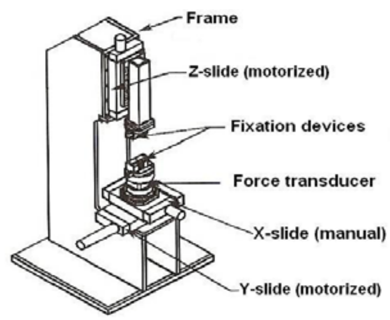

Fig.2. The DISC instrument, the tooling is mounted on the force transducer

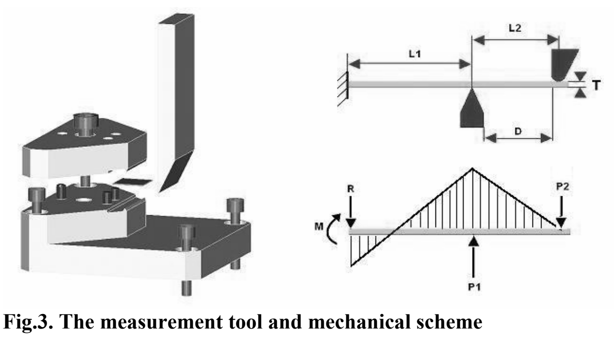

Fig.3. The measurement tool and mechanical scheme

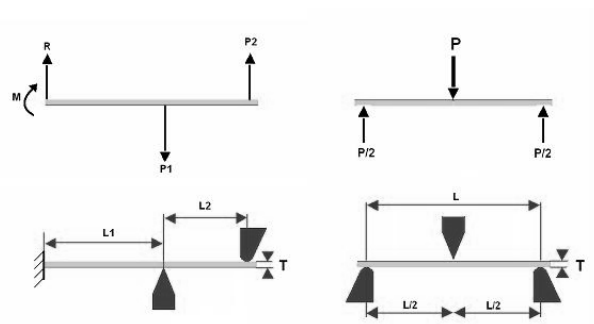

Fig 4. Comparison between mechanical schemes of the PD-test (at the left) and the 3-P bend test (at the right)

Carrying out the bending test as described above is simple and requires only low forces (< 100 N) compared to the tensile test, additionally it can be done under any angle to the rolling direction, and with samples taken from narrow strips of the type used to produce electrical contacts.

3. Methods of transforming force-deflection curves to stress-strain curves

The output curve of a PD bend test measurement is shown graphically in figure 5, four zones are distinguished. The best defined zones are zone 2 and zone 4. In zone 2 the behaviour is fully elastic and the force develops linear with the stresses everywhere in the sample. Here the E-modulus, the sample geometry and boundary conditions determine the slope of the linear part. In zone 4 the force is limited by the behaviour of the cross-section with the highest load, near the pivot point. At the maximum force the stress distribution above the pivot is nearly rectangular and is supposed to be completely rectangular for the sake of calculation. In reality the stress near the surface must be somewhat higher because a neutral zone still exists in the middle and there must be some effect from strain hardening.
A short discussion of each zone follows underneath.

Zone 1 is a run-in zone where the build-up of force as a function of deflection depends on how parallel and flat the sample and the tooling are. This part of the curve will be corrected to be linear and to pass through the origin by extrapolation downwards from the linear part in zone 2 and making the deflection zero at force zero. The force build up in zone 1 is usually less than 10% of the measured force maximum.

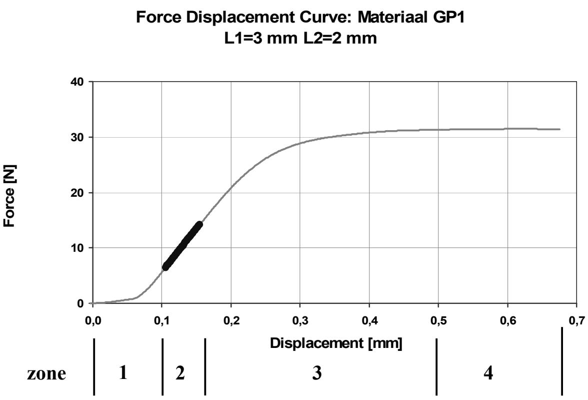

Fig. 5. Example of a measured force-deflection curve, the curve is divided into 4 zones along the horizontal axis.

Zone 2 is a nearly linear part with a slope that reflects the elastic stiffness of the particular sample and measuring configuration. The behaviour is fully elastic and the stresses develop proportional to the force. The E-modulus, the sample geometry and boundary conditions determine the slope of the linear part.

The E-Modulus can be approached using the theory and formulas from the mechanics of prismatic elastic beams. The length L_2 and the material thickness are in the results the most critical parameters as they are in the third power. The linearity in the interval from 20% to 45% of the measured maximum force is quite good (the trend line has a R^2 better than 0,999), though generally somewhat less straight in rolling direction than transverse. This is demonstrated in figure 6 in a diagram of secant modulus versus strain showing how the value of the E-modulus for solid solution hardened material G-P1 depends on which interval is chosen for its determination, while it is well defined for precipitation hardened sample I-T1.

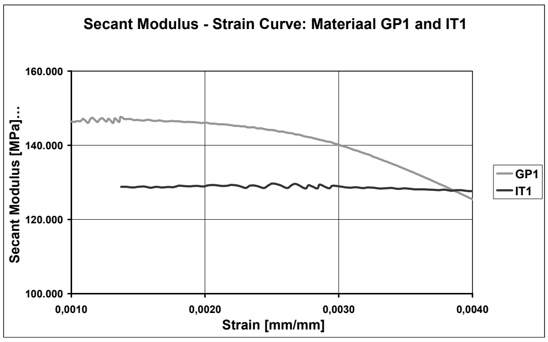

Fig. 6. Calculating the slope from lines from the origin (0,0) to each point of the bending curve produces a diagram of the secant modulus as a function of strain. Material I shows a well defined E-modulus, the value for material G will vary with the selected interval of the curve.

The deflection needs to be corrected for the compliance of the measurement set-up. This correction is done by using a stiff part to measure the compliance of the set-up, and then subtract the deflection of the set-up from the measured deflection at each force value.
The formula to calculate the E-Modulus is for the PD-method as follows:

$$\mathbf{E= P_2*(4*L_2^3+3L_1*L_2^2)/((d-d_{corr})*W*T^3)} \qquad \{1\}$$

with E= E-Modulus, P_2 the measured force, L_1 en L_2 the distances as in figure 1, d the measured deflection, d_{corr} the correction of the deflection for the stiffness of the instrument, W the width of the sample and T the thickness of the sample.
For the 3P-method the formula is as follows:

$$\mathbf{E = P*L^3/((d-d_{corr})*4*W*T^3)} \qquad \{2\}$$

Herein are P and L as in figure 1 and other symbols as above with the PD-method.

Zone 3 is the transition from the linear zone 2 to the flat zone 4. The deformation is still elastic in the larger part of the sample but no longer so in or near the cross-section with the highest stress near the pivot point where plastic yielding occurs. This plastic yielding area grows until the cross-section with the highest stress reaches the maximum load it can bear; thereafter the force and the stress do not increase any more. The shape of the stress distribution in the cross-section with the highest stresses approaches a rectangular form and determines the maximum force level: it becomes a plastic hinge. The theory is illustrated in figure 7.
The stress in the formula for plastic deformation is as a function of moment 2/3 the stress calculated as if it had been elastic. This multiplied by the factor ½√3 to transform the bending stress into the von Mises stress produces a factor 1/3√3. The transformation from a force-deflection curve to a stress-strain curve can now be done by introducing a factor 1/3√3 in the linear formulas for stress **and** strain. Doing so delivers the proper slope in zone 2 and the proper stress maximum in zone 4 and a gradual transition which is likely to be on the conservative side as the factor 1/3√3 is used in the region where the cross section is not completely plastic.

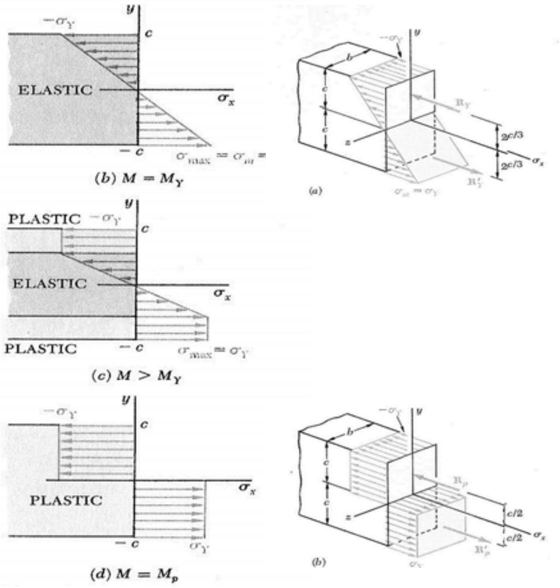

Fig. 7. Stress and strain diagrams from a cantilever beam elastically and plastically deformed. At the top is shown the distribution of elastic stress (zone 2), in the middle the elasto-plastic situation (zone 3) and below the stress during plastic deformation in the cross-section with the highest load.

The formulas for the elastic stress and strain are:

$$\boldsymbol{\sigma} \mathbf{= 6*P_2 *L_2/(W*T^2)} \qquad \{3\}$$

$$\boldsymbol{\varepsilon} = \boldsymbol{\sigma}\mathbf{/E=6*(d-d_{corr})*T/(4L_2^2 + 3*L_1*L_2)} \qquad \{4\}$$

For maximum stress in plastic deformation, which is with the PD-method in the cross-section near the pivot, they become (by multiplying with the factor 1/3√3):

$$\boldsymbol{\sigma}_{\mathbf{vonMises}} \mathbf{= 2\sqrt{3}*P_2*L_2/(W*T^2)} \qquad \{5\}$$

$$\boldsymbol{\varepsilon}_{\mathbf{pd}} \mathbf{= 2\sqrt{3}*(d-d_{corr})*T/(4L_2^2 + 3*L_1*L_2)} \qquad \{6\}$$

And with the 3P-method in the middle:

$$\boldsymbol{\sigma}_{\mathbf{vonMises}} \mathbf{= (2/3)\ \sqrt{3}*P*L/(W*T^2)} \qquad \{7\}$$

$$\boldsymbol{\varepsilon}_{\mathbf{pd}} \mathbf{= 2\sqrt{3}*(d-d_{corr})*T/L^2} \qquad \{8\}$$

Zone 4 is the part of the curve where the force does not increase any more as a function of deflection. It is important to choose the length between the locations of the load and of the fixation in dependence of the thickness of the sample, so that the effect of the change in shape of the sample can be neglected.
This can be done by estimating the maximum allowable length with a formula that expresses the maximum

length as a function of the maximum angle in the sample. In order to estimate the distance in advance the following formula can be used for the PD-method:

$$L_2 = (2/3)*(E/ \sigma_{vonMises, max})*T*\varphi_{max} - 0.5*L_1 \qquad \{9\}$$

For the 3P-method:

$$L = (4/3)*(E/ \sigma_{vonMises, max})* T*\varphi_{max} \qquad \{10\}$$

I recommend keeping the angle φ_{max} at less than approximately 10 degrees or 0.175 radians. The formulas are based on the rectangular, plastic stress distribution to make sure that φ_{max} is not underestimated. Note that the formulas offer an estimate for a setting for the initial length adjustment based on figures that are actually still to be measured. For σ_{max} and **E** the data from material suppliers can be used as a first estimate. When the sample gets a too large angle the measurement should be repeated using measured data.

4. Measurement of the Poisson constant

The described material measurement delivers the data for the case of pure bending. Many contact springs have a combination of bending and torsion.
In torsion the shear modulus G is the material constant comparable to the modulus of elasticity E in tension.
The well known relation between E and G is as follows:

$$E = 2*G*(1 + \nu) \qquad \{11\}$$

Herein is ν the Poisson constant. Finite element programs need a value of the Poisson constant for correct prediction of torsion. Therefore the input of a correct value for the Poisson ration can be important.
To my knowledge there is no easy way to measure the Poisson constant of contact materials.
It is not difficult to change the bending experiment into a mixed bending/torsion experiment by applying the load on the edge of the sample by putting the actuator under an angle, see figure 8. To estimate the effect of the Poison constant a finite element simulation has been performed with ν = 0.25, 0.3 and 0.35. The resulting reaction forces with the particular sample have been 22.8, 24.9 and 27.6 N, so a difference of 20%.

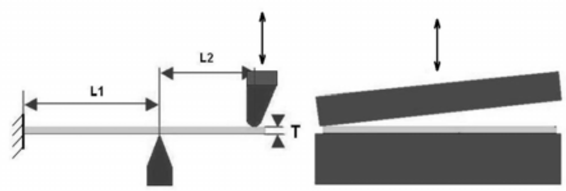

Fig. 8. The proposed set-up for the measurement of the Poisson constant.

Combining the mixed bending/torsion experiment with the pure bending experiment and a finite element simulation for both cases is a relative simple way to establish the Poisson constant from small samples and also under any angle with the rolling direction. No experiments have been conducted yet.

5. Measurement results

The measurements have been done with 17 materials.
The result for 4 selected representative materials, they are listed in table 1, will be used to show the results. CT, FT and IT stand for materials C, F and I in Transverse direction, the 1 in C-T1 means the first of the three measurements. Similarly is G-P1 the 1st measurement with material G in Parallel direction.

code	Supplier Code	Composition	thickness specified	thickness measured
			mm	mm
C-T	Stol 94 R750	CuNi2,6Si0.6Sn0.7Zn0.8	0,150	0,152
F-T	Argeste 1.4310	X10CrNi18-8	0,140	0,140
G-P	B14	CuSn4	0,190	0,194
I-T	K57 TM04	CuNi1CoSi	0,150	0,150

Table 1. The materials selected for this publication. CT, FT and IT are materials C, F and I, measured in transverse direction. GP is material G in parallel direction.

Two different measurements have been carried out:

- Measurements at low force to establish the E-Modulus in the elastic region, referred to as **E-measurements**. Each material has been measured twice independently, both in the rolling direction and transverse. Values for the E-Modulus and for the maximum stress are averages from these two values.
- Measurements up to the ultimate strength into the plastic region like shown in figure 5, referred to as **elastoplastic** measurements. Each material has been measured three times independently, both in the rolling direction and transverse. Values for the E-Modulus and for the maximum stress are averages from these three values.

The reduction of the distance L_1 from 4.5 to either 1, 2 or 3 mm has improved the quality of the elastoplastic measurements considerably, especially for the very elastic materials. The measurement of material F benefits particularly from this change and gives with L_1=L_2=1 mm higher values for stiffness and strength than in previous measurements with L_1=4.5 mm.
It appeared in the measurements that two factors caused errors. The first factor is the sensitivity for alignment and positioning of the actuator. Even deviations in L_2 of 10 µm cause with L_2=1 mm 3% deviation in the stiffness results. It happened that a slight rotation of the lower part of the set-up went unnoticed, causing deviations in the results of 5-10%.
The second factor, explained before, is that the figure for the E-modulus is strongly influenced for some materials by the length chosen as being the straight area in zone 2.
Table 2 shows for the 4 selected materials from left to right the values from the E-measurements, the elastoplastic measurements, from previous E-measurements and supplier specified values.

	2008	2008	2006	
	E-modulus measurement	Elastoplastic measurement	E-modulus measurement	supplier spec
	GPa	GPa	GPa	GPa
C-T	158	155	152	132
F-T	218	204	213	193
G-P	133	144	129	120
I-T	124	127	119	131

Table 2. From left to right the values from the recent E-measurements and elastoplastic measurements, from previous E-measurements from 2006 and supplier specified values.

Values from E-measurements tend to be higher than from elastoplastic measurements because an interval at a lower force is used. A higher figure in elastoplastic measurements indicates a deviation in L_2.

Figure 9 shows a measurement result with material C-T1 as an example. The measurement is the black, lower curve. The light grey curve in the middle is the measurement curve corrected for the stiffness of the measurement set-up; it includes a darker section that shows which part has been used to establish the E-modulus. The grey upper curve is extrapolated downwards and moved to the point zero (0,0).

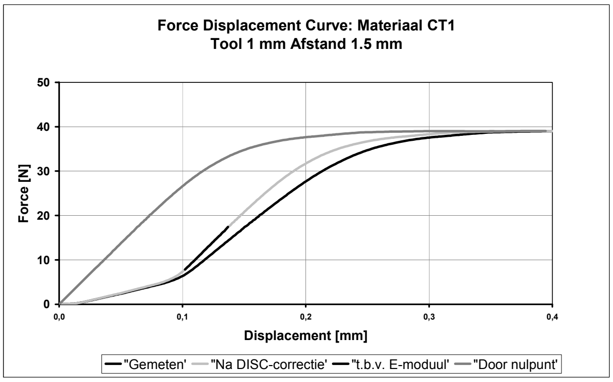

Fig. 9. The measurement curve (black) and illustration of changes to obtain the corrected upper curve.

Figure 10 shows the corrected force-deflection curves for the 4 materials. Application of the formulas {5} and {6} to these graphs result in the stress-strain graphs in figure 11.

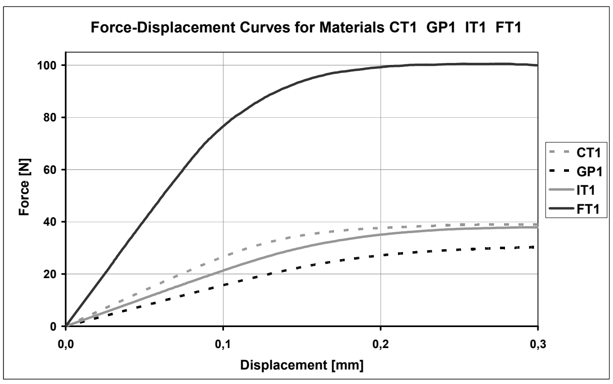

Fig. 10. Force-Deflection curves as measured and corrected.

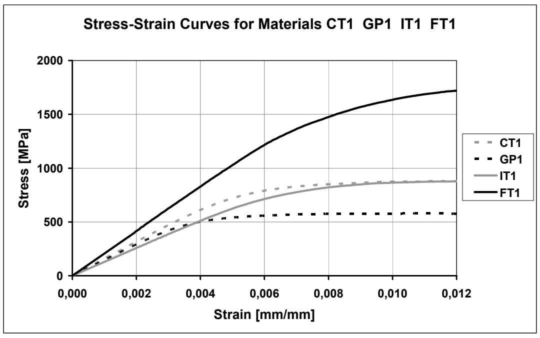

Fig. 11. Stress-Strain curves derived from the Force-Deflection curves from figure 10 with formulas {5} and {6}.

The ultimate strength values, derived from the maximum force with formula {5}, are compared in table 3. Most values agree within 3% compared to the previous results from 2006, the larger differences are increases due to the effect of the tool improvement. This is the case with materials F and I.

	2008	2006	supplier specified
	$\sigma_{von\ Mises,max}$	$\sigma_{von\ Mises,max}$	σ_{uts}
	MPa	MPa	MPa
C-T	861	850	760
F-T	1768	1572	1710
G-P	569	561	610
I-T	861	824	800

Table 3. Ultimate strength-results for the 4 materials. From left to right columns with recent values, values from previous investigation (corrected to $\sigma_{vonMises,max}$) and supplier specified values.

6. Finite element calculation results

Finite element simulations have been done to evaluate the different ways of deriving input for FEM-simulations from the stress-strain curve. The idea is to use the data derived from the force-deflection curves as input for the finite element analysis and then compare the force-deflection curves from the simulation with the originally measured curve.

The following alternatives are considered:

(a) a bilinear curve using the measured E-Modulus and a horizontal line with $\sigma_{vonMises,\ max}$ as maximum stress.

(b) a bilinear curve using the measured E-Modulus with a line from $\sigma_{vonMises,\ 0.2}$ through the $\sigma_{vonMises,\ max}$ point.

(c) a multilinear characteristic closely following the derived stress-strain curve

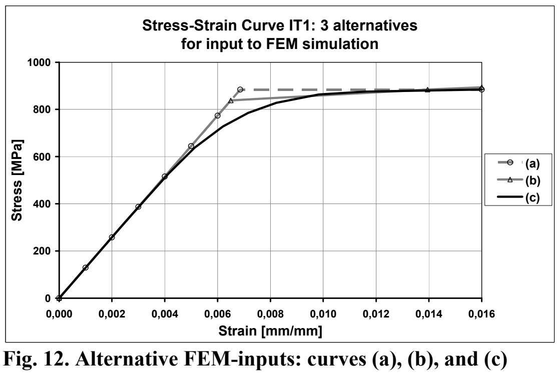

Fig. 12. Alternative FEM-inputs: curves (a), (b), and (c)

It appeared that analysis times with multi-linear curves (c) are much longer than with bilinear curves (a) and (b) and result in nearly identical curves as can be seen in figure 13, which shows the measured and simulated curves for material C.

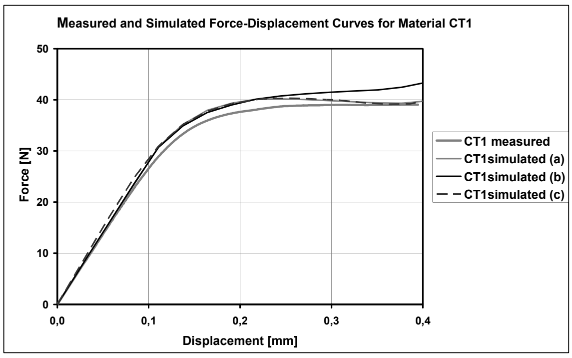

Fig. 13. The measured curve and simulations with bilinear curves (a) and (b) and multilinear curve (c).

A good fit between measurement and simulation indicates that the material data are realistic, also for simulation of a more complicated geometry. For practical purposes it is desirable to keep the approach as simple as possible while being on the safe side. From the first results it appears that simplest alternative (a) gives the best fit, however none of the curves are at the safe side. Some safety factor may be desirable in order to take into account a variability of batches and product tolerances.

7. Conclusions

Formulas are presented to derive stress strain curves directly from the measured force deflection curves; finite element analyses confirm the validity of the approach.

The measurement method provides a simple way to produce the data needed as input for finite element simulation, using small samples of original strip material intended for use in manufacturing the particular springs.

The lengths and the material thickness need to be measured with µm accuracy; they are in the test the most critical parameters as they are in the third power.

The PD method and the 3P method can both be used; the preference for the PD method is because of easier alignment and handling in our laboratory.

With several materials the figure for the E-modulus depends strongly on the selected interval where it is determined

The primary interest in this method is for manufacturers and designers of contact springs, more generally of springs produced from thin rolled metal strip.

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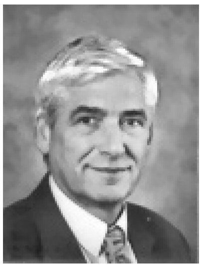

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